The Perception and Cognition of Pitch Structure in Tonal Music

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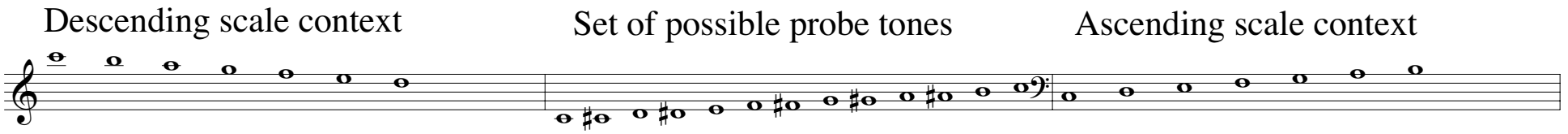
1. Krumhansl's (1990) studies on the perception and cognition of pitch relationships in Western tonal music

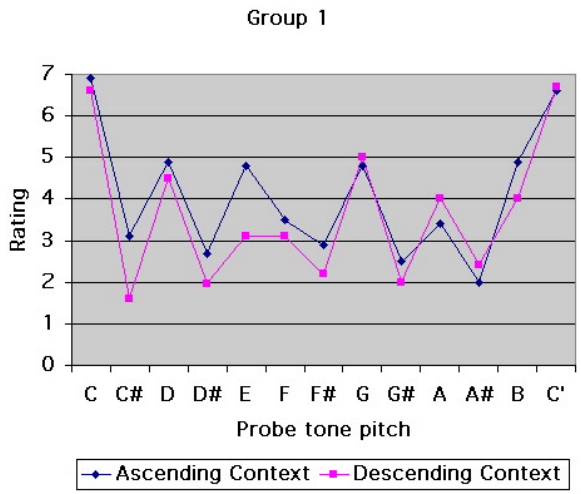
	Tones	*Chords*	*Keys*
Tones	Krumhansl (1990, Chapter 5)	-	-
Chords	Krumhansl (1990, Chapter 7)	Krumhansl, Bharucha, and Kessler (1982b); Bharucha and Krumhansl (1983); Krumhansl, Bharucha, and Castellano (1982a); Krumhansl (1990, Chapter 8)	-
Keys	Krumhansl and Shepard (1979); Krumhansl and Kessler (1982); Krumhansl (1990, Chapter 2)	Krumhansl (1990, Chapter 7)	Krumhansl and Kessler (1982); Krumhansl (1990, Chapters 2 and 7)

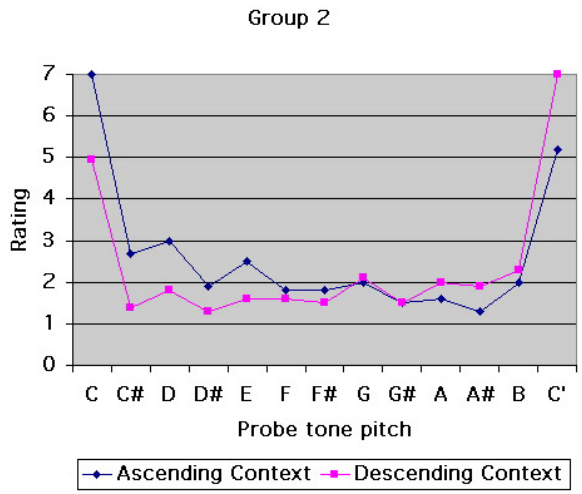
1. Krumhansl's (1990) studies on the perception and cognition of pitch relationships in Western tonal music

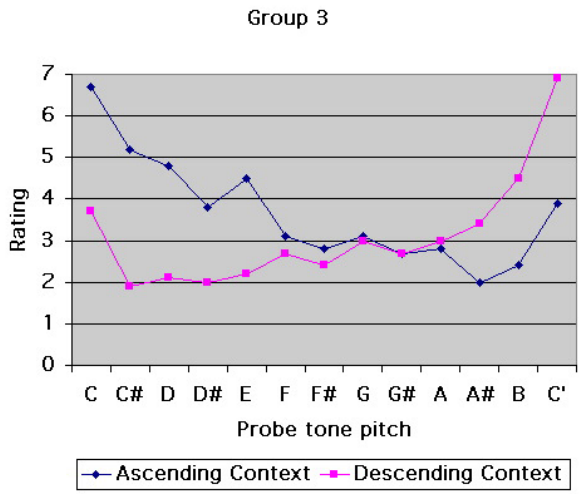
1. In her book, *Cognitive Foundations of Musical Pitch*, Krumhansl (1990) reviews the experimental studies that she carried out with various collaborators in which she explored the perception of relationships between tones, chords and keys in Western tonal music.
2. The goal of these experiments was to characterize "the listener's internal system for processing pitch structures, focusing primarily on those found in traditional Western music" (Krumhansl, 1990, p. 9).
3. This table shows the type of perceptual relationship investigated in each of the experiments that she reviews in her book.
4. Most of these experiments took the form of "probe-tone studies".
5. In a probe-tone study, a listener first hears a standard pattern called the 'context' and then hears a tone called the 'probe-tone'. The listener then has to rate, usually on a scale of 1 to 7, how well the probe-tone fits with the context.

2. Krumhansl and Shepard (1979): Investigating the perceived relationships between tones and keys









2. Krumhansl and Shepard (1979): Investigating the perceived relationships between tones and keys

1. The first two experiments to use the probe-tone method were those described by Krumhansl and Shepard (1979).
2. In these experiments, listeners were presented with incomplete ascending or descending C major scales, followed by probe tones with various pitches. The listeners had to judge how well each probe tone completed the incomplete scale given as a context.
3. In the first experiment, the incomplete descending scale used was this one shown here on the left and the incomplete ascending scale used was the one here on the right.
4. Each probe tone was chosen from the set of 13 tones in the chromatic scale starting on middle C and ending on the C an octave above, as shown here in the middle.
5. In the second experiment, Krumhansl and Shepard (1979) added the quarter tones in between these chromatic scale tones to the set of probe tones used.
6. These graphs summarize the results that they obtained.
7. The graph here on the left shows the results obtained for the musically most experienced subjects who had had an average of 7.4 years of musical training; the middle graph shows the results obtained for subjects with around 5.5 years of musical training; and the graph here on the right shows the results obtained for subjects with less than a year of musical training.
8. For the most highly trained subjects,
   (a) the results obtained for the ascending context were very similar to those obtained for the descending context;
   (b) the two tonic tones received the highest rating and both tones received an approximately equal rating; and
   (c) the tones in the C major scale received a higher rating than the tones outside the C major scale.
9. For the group with a moderate amount of musical training,

(a) again, the ratings for the tonic tones were a lot higher than the ratings for the other tones but the rating for the tonic furthest away from the context was lower than that for the tonic nearest in pitch to the context;

(b) apart from the tonic tones, there was a gradual decrease in the rating as the tones became more and more distant in pitch from the last tone of the context; and

(c) again, the scale tones received higher ratings than the non-scale tones but this time the difference between the ratings for the scale tones and those for the non-scale tones was not as great as for the most experienced subjects.

10. For the group of subjects with the least training,

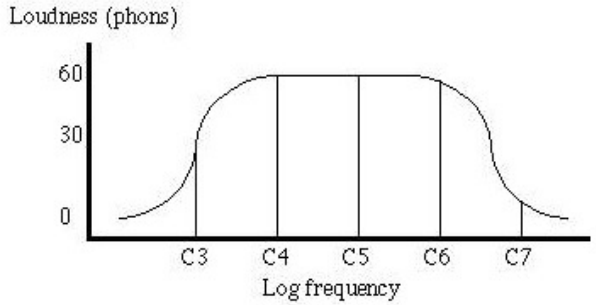
(a) tones further away from the end of the context were generally rated lower;

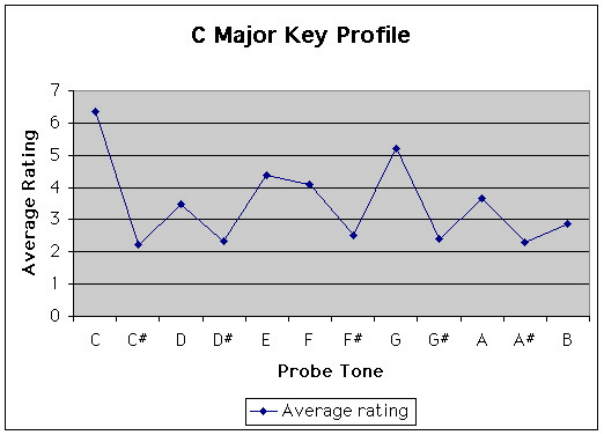
(b) the tonic furthest from the context was rated higher than would be predicted by its distance from the context, but not as much higher as it was by the subjects with more training; and

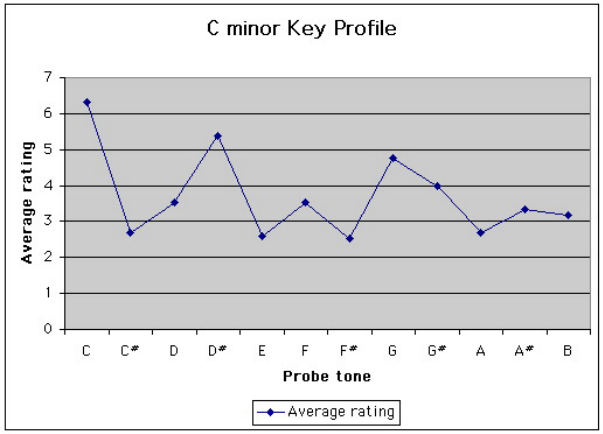
(c) scale-tones were not rated significantly more highly than non-scale tones.

11. These results suggest that, with training, Western listeners acquire what Krumhansl calls a *tonal hierarchy* in which one tone, the tonic, is judged to be most stable, while the other scale-tones are judged less stable but more stable than those tones outside of the scale.

3. Krumhansl and Kessler (1982): Investigating the relationships between tones and keys







- Used various different major and minor key-defining contexts.
- Used various different tonics.
- Subjects given practice trials and told to make full use of rating scale.
- Subjects made multiple judgements of the same context-probe-tone combination.
- Only subjects with at least 5 years of musical training used.
- Used circular tones to eliminate effect of pitch distance.
- Subjects asked to judge fittingness rather than completion.

3. Krumhansl and Kessler (1982): Investigating the relationships between tones and keys

1. Krumhansl and Kessler (1982) replicated and extended the experiments of Krumhansl and Shepard (1979).
2. Krumhansl and Kessler (1982) used various different key-defining contexts to test the hypothesis that any strongly key-defining context will give rise to similar probe-tone ratings.
3. Used major and minor key contexts.
4. Used various different tonics to test the hypothesis that a similarly shaped tonal hierarchy would be obtained for any tonic.
5. Used a more careful experimental procedure in which the subjects were given some practice trials before starting the experiment proper, and instructed to use the full range of the rating scale.
6. The trials were arranged in blocks so that listeners made multiple judgements for each probe-tone-context combination. Within each block, the order of the trials was randomized. The order of the blocks was randomly selected for each subject.
7. To eliminate the possibility of subjects adopting an intentionally unmusical strategy in their responses (which might have been the case for the non-musicians in the experiments of Krumhansl and Shepard (1979)), only subjects with at least 5 years of formal musical training were chosen.
8. Krumhansl and Shepard (1979) had used flute tones and sine waves, both of which have very definite qualities of tone *height*. That is, given two flute tones or two sine waves that are a semitone or more apart, two subjects will generally agree about whether the first tone is higher than the second or vice-versa.
9. It was clear from the results of Krumhansl and Shepard's (1979) experiments that listeners with less musical training tended to give lower ratings to tones that were further away from the context.
10. Krumhansl and Kessler (1982) attempted to eliminate the effect of pitch distance by using special tones called 'circular tones' that are not perceived to have any definite sense of pitch height.

11. Each of the circular tones use by Krumhansl and Kessler (1982) contained five sine-wave components, each one an octave above the next, the amplitudes of these components being fixed so that the spectrum of the tone had a bell-shape as shown here.

12. Because the highest and lowest components of these tones have very low amplitudes, the tones have an organ-like quality with no well-defined lowest or highest pitch and approximately the same overall pitch height.

13. One other difference between the experiments of Krumhansl and Kessler (1982) and those of Krumhansl and Shepard (1979) was that the contexts used by Krumhansl and Kessler (1982) sounded complete and the subjects were asked to judge how well the probe-tones fit with the context, rather than how well the probe-tones *completed* the context.

14. The results of the experiment showed that the subjects agreed very well with each other and gave similar responses each time a particular context-probe-tone combination was heard.

15. They also found that essentially the same shape of profile was obtained for all major keys and a different profile was obtained for all the minor keys.

16. They also found that essentially the same responses were obtained regardless of the specific key-defining context used.

17. The graph here on the left shows the rating profile obtained for the major key context and the graph here on the right shows the profile obtained for the minor key context. Both graphs are drawn as though the tonic is C.

18. For the major key context,

    (a) the highest rating was given to the tonic;

    (b) the next highest rating was given to the dominant;

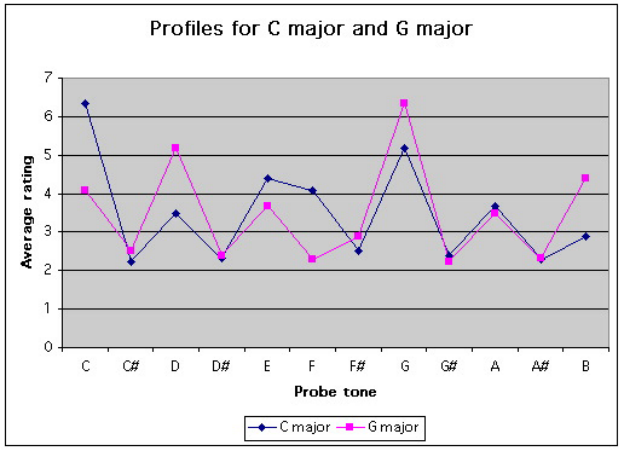
    (c) followed by the mediant;

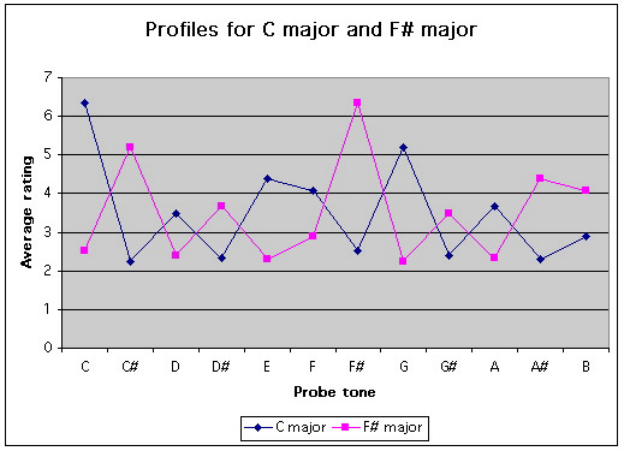
    (d) then the other scale tones; and finally

    (e) the non-scale tones.

19. For the minor key context,
    (a) the highest rating again given to the tonic;
    (b) followed by the mediant and the dominant;
    (c) then the other scale degrees; and finally
    (d) the non-scale degrees.

4. Krumhansl and Kessler (1982): Measuring the perceptual distance between keys





f# d bb

Db/C# A F Db/C#

c# a f

E C Ab

g# e c

B G Eb

eb/d# b g eb/d#

F# D Bb

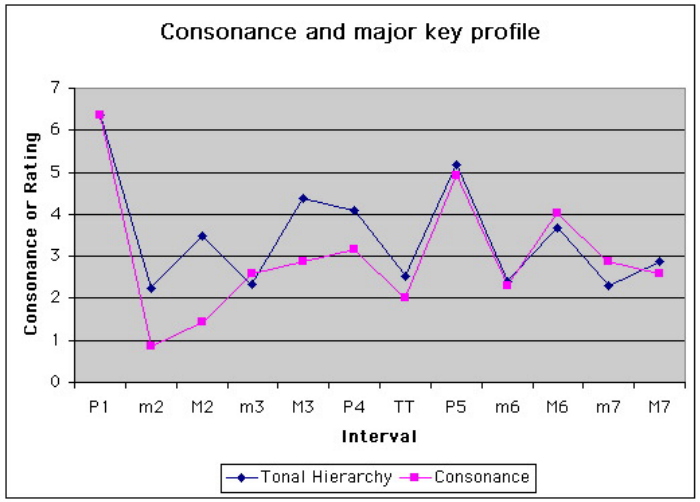
f# d bb

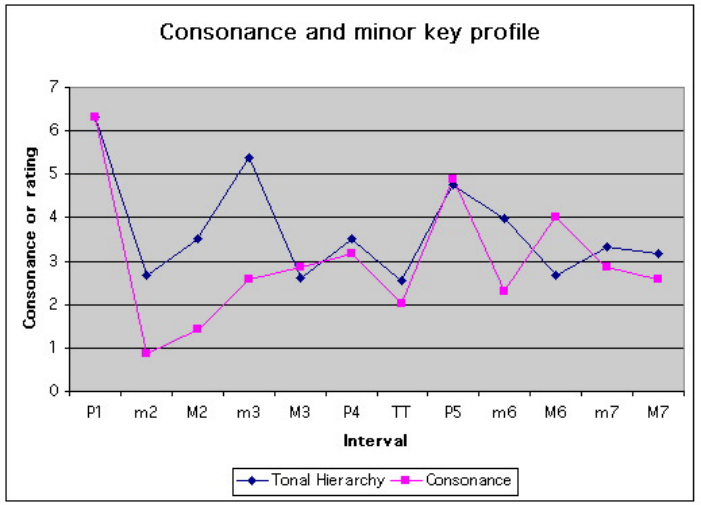
4. Krumhansl and Kessler (1982): Measuring the perceptual distance between keys

1. Each of the probe-tone rating profiles or 'tonal hierarchies' obtained by Krumhansl and Kessler (1982) describes the perceived pattern of stability imposed on the tones within an octave by a particular key.
2. In tonal music theory, modulations generally occur between keys that are considered 'closely related'.
3. In general, a major key is considered to be closely related to the major key of its dominant, the major key of its sub-dominant, its relative minor and its tonic minor.
4. While a minor key is considered to be closely related to the major or minor key of its dominant and subdominant, and its relative and tonic major keys.
5. Krumhansl and Kessler (1982) proposed that the perceived relatedness of two keys could be predicted by measuring the similarity of their probe-tone rating profiles.
6. On the previous slide I showed you the probe-tone rating profiles for C major and C minor. The probe-tone rating profiles for the other keys are the same shape as these profiles except that they begin on different pitch classes. For example, the rating profile for G major would be equal to the profile for C major shifted along by seven semitones, as shown in this graph here on the left.
7. As you can see from this graph, the rating profiles of C major and G major are quite similar—they generally go up and down in the same places.
8. On the other hand, this graph shows the rating profiles of C major and F♯ major which are quite different, as you can see—where one profile goes up, the other tends to go down.
9. One standard statistical measure of the degree of similarity between two sets of data is called the correlation.
10. The correlation between two sets of data is a value between -1 and 1. A correlation of 1 means that the two sets of data are exactly the same, and a correlation of -1 means that they are exactly opposite.
11. The correlation between the C major profile and the G major profile is .651 which is a fairly high value.

12. The correlation between the C major profile and the F♯ major profile is -.683 indicating a fairly high anti-correlation.
13. After finding the correlation between every pair of keys, Krumhansl and Kessler (1982) used a technique called *multidimensional scaling* to get a spatial representation in which each key is represented as a point and the correlation between any two keys is represented by the distance between the two points representing the keys.
14. They found that they could only represent the correlation between the keys in this way if they used a four-dimensional Euclidean space, that is, a space in which the distance between any two points is measured in a straight line.
15. However, they found that if they used a curved space in which the inter-key distances were measured within the curved surface of a torus they could represent the inter-key distances in three dimensions.
16. This diagram here gives a visual representation of the correlations between the keys. In this diagram, the torus has been cut along both of its circumferences and unfolded. [Demonstrate with piece of paper.]
17. Note that in this multidimensional scaling solution to the results of their experiment, the circle of fifths emerges as a 'straight line' within the surface of the torus and the relative major/relative minor pairs are also close to each-other.

5. Correlation between tonal hierarchies and tonal consonance





- *Tonal consonance:* Psychophysical phenomenon in which two tones sounding simultaneously sound 'fused', 'smooth', harmonious or 'pleasing'.
- *Musical consonance:* Two tones sounding simultaneously interpreted to be stable in a particular musical style.
- Various different measures of tonal consonance proposed (e.g., Helmholtz, 1954; Kameoka and Kuriyagawa, 1969; Malmberg, 1918)
- Quite high correlation between major key profile and Helmholtz's (1954) measures of consonance.
- Lower, but still statistically significant correlation between minor key profile and consonance measure of Kameoka and Kuriyagawa (1969).

5. Correlation between tonal hierarchies and tonal consonance

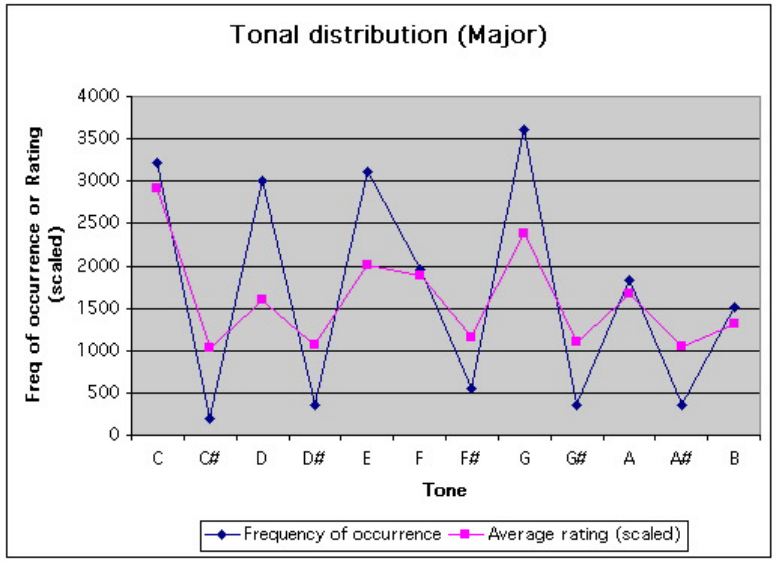
1. It's important to distinguish between *tonal consonance* and *musical consonance*.
2. The degree of tonal consonance of a pair of tones sounding together is supposed to be a measure of how much they fuse into a single percept or how harmonious they sound.
3. On the other hand, the musical consonance of a given pair of tones sounding simultaneously is determined by how stable they are interpreted to be in a particular musical style.
4. Thus, for example, the perfect fourth is considered musically dissonant in a tonal context when heard between the lowest part and a higher one.
5. However, according to most measures of tonal consonance, the perfect fourth is a highly consonant interval.
6. So, although there is obviously some connection between tonal consonance and the intervals that have evolved to be musically consonant in certain musical styles, there is no direct equivalence between the two forms of consonance.
7. Various different measures of tonal consonance have been proposed since the 12th century. The most well-known is probably the hypothesis that tones whose fundamentals frequencies are separated by low-integer ratios are heard to be consonant, the higher the integers involved in the ratio, the more dissonant the interval is heard to be. This measure was originally proposed by Pythagoras.
8. Helmholtz (1954) noticed that when two tones are close but not identical in pitch, a sensation of beating or roughness is heard.
9. More modern studies (see Rasch and Plomp, 1999, pp. 106–108) explain this sensation of roughness by claiming that when the sine wave components of complex tones are close enough together, they 'interfere' with each other by stimulating the same area of the basilar membrane. This occurs when two sine-wave components are within what is called one 'critical bandwidth' of each other.
10. Krumhansl (1990, pp. 55–62) calculated the correlations between the tonal hierarchies and various different tonal consonance measures.

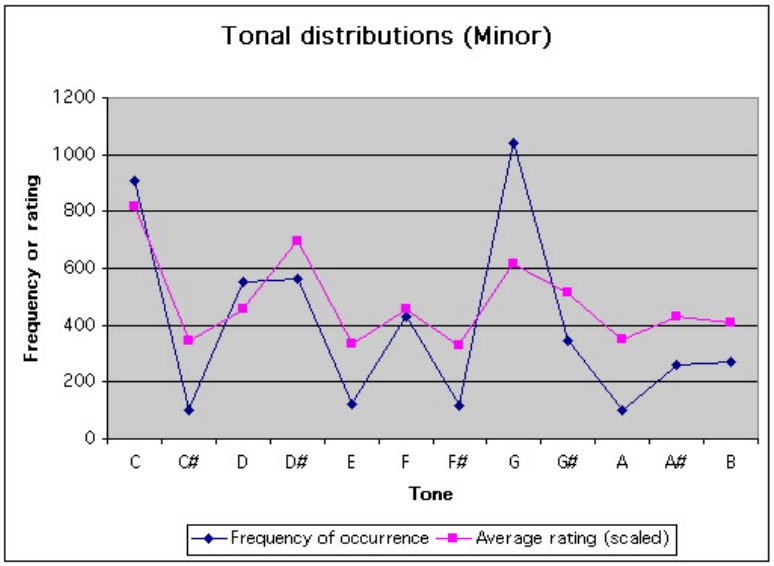
11. She found that there was a statistically significant correlation between the major key probe-tone profile and a number of different measures of tonal consonance; and a somewhat lower, but still significant correlation between the minor key probe-tone profile and one particular measure of tonal consonance given by Kameoka and Kuriyagawa (1969).

12. These two graphs show the consonance measures of Kameoka and Kuriyagawa (1969) plotted against the average probe-tone ratings obtained by Krumhansl and Kessler (1982).

13. You can see from these graphs that the correlation between consonance and the major key profile is considerably better than that between consonance and the minor key profile.

6. Correlation between tonal hierarchies and statistical distribution of tones in tonal works



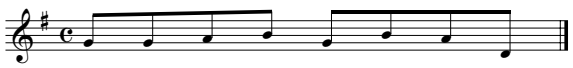


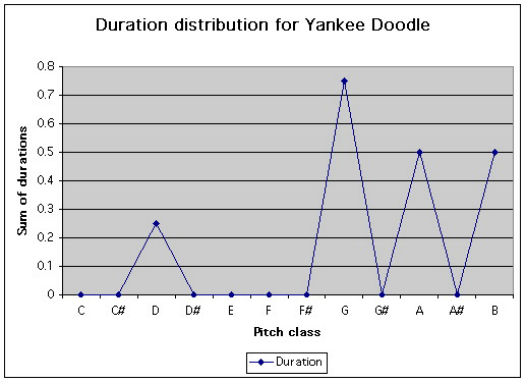
- Youngblood (1958) and Knopoff and Hutchinson (1983) measured distribution of tones in songs and other vocal works by Schubert and others.
- Krumhansl (1990) found very high correlations between frequency distributions of tones in these works and key profiles.

6. Correlation between tonal hierarchies and statistical distribution of tones in tonal works

1. Youngblood (1958) and Knopoff and Hutchinson (1983) measured the frequency of occurrence of each of the 12 chromatic scale tones in various songs and other vocal works by Schubert, Mendelssohn, Schumann, Mozart, Richard Strauss and J. A. Hasse.
2. The graph on the left here shows the frequencies of occurrence of tones in major key works and the graph on the right shows the frequencies of occurrence of tones in minor key works.
3. The correlation between the major key profile and the distribution of tones in major works is considerably higher than the correlation between tonal consonance and the major key profile (and similarly for the minor key profile).

7. Krumhansl and Schmuckler's (1990) key-finding algorithm





Key	Score	Key	Score
C major	0.274	C minor	-0.013
C sharp major	-0.559	C sharp minor	-0.332
D major	0.543	D minor	0.149
E flat major	-0.130	E flat minor	-0.398
E major	-0.001	E minor	0.447
F major	0.003	F minor	-0.431
F sharp major	-0.381	F sharp minor	0.012
G major	0.777	G minor	0.443
A flat major	-0.487	A flat minor	-0.106
A major	0.177	A minor	0.251
B flat major	-0.146	B flat minor	-0.513
B major	-0.069	B minor	0.491

7. Krumhansl and Schmuckler's (1990) key-finding algorithm

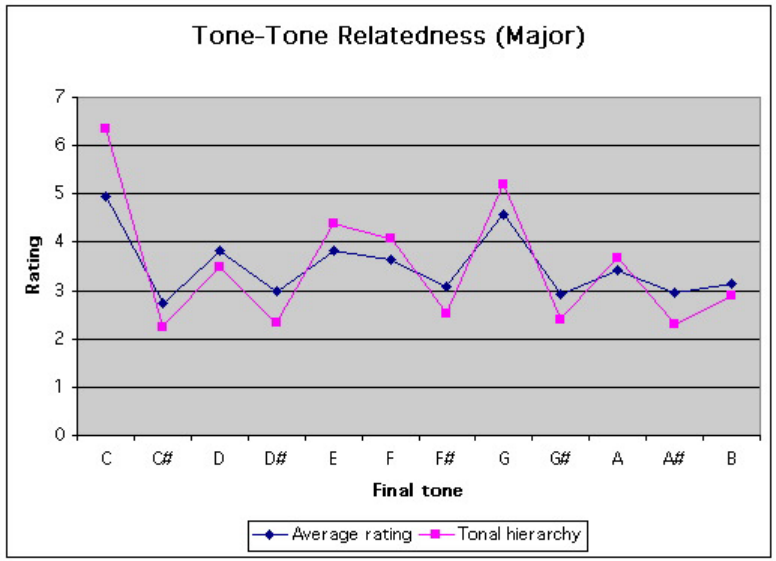
1. Krumhansl (1990, p. 77) takes the strong correlation between the frequency distribution of tones in tonal works and the probe-tone key-profiles to suggest that "the tonal hierarchies might be acquired through experience with the musical style, particularly through internalizing the relative frequencies and durations with which tones are sounded".
2. Krumhansl and Schmuckler (1990) propose a key-finding algorithm based on tonal hierarchies.
3. This algorithm takes a representation as input that gives the duration and chromatic pitch (i.e., MIDI note number) of each note in the passage and generates as output a prediction of the key that the passage will be perceived to be in.
4. The first step in the algorithm is to count up for each different pitch class the sum of the durations of tones with that pitch class in the passage.
5. For example, if we take the opening bar of 'Yankee Doodle', as shown here, we find that
   (a) the sum of the durations of the G naturals gives .75 of a minim,
   (b) the durations of the B naturals add up to half a minim,
   (c) the durations of the A naturals add up to half a minim
   (d) and there is one quaver D natural.
6. We can then draw a graph showing the durations of the various pitch classes within the passage being analysed, as shown here.
7. The next step in the algorithm is to calculate the correlation between this graph and each of the 24 major and minor key profiles. This table here shows the correlation between this graph showing the durations of the various pitches in the 'Yankee Doodle' excerpt and each of the major and minor key profiles.
8. The algorithm then predicts that the perceived key will be the one whose profile best correlates with the graph showing the distribution of tone durations for the passage.

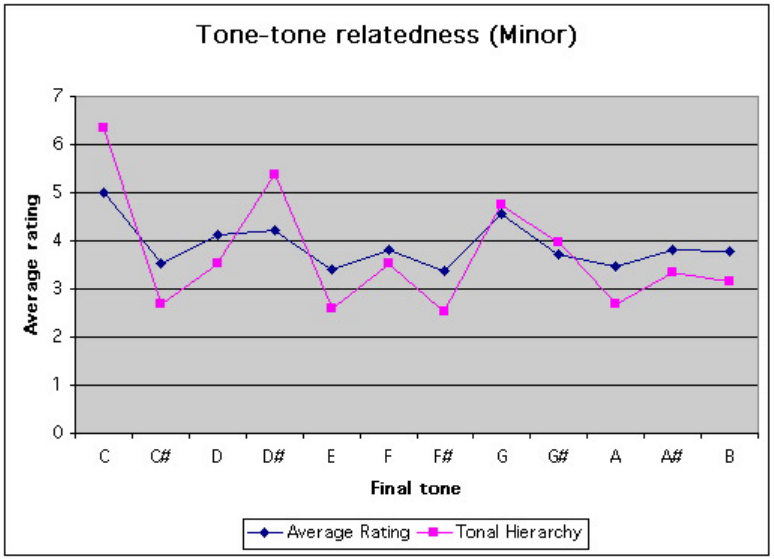
9. So in this case, the algorithm correctly predicts that the key of 'Yankee Doodle' is G major.

10. Krumhansl (1990, p. 84) applied the algorithm to the first four tones of each of the Preludes in Bach's *Das Wohltemperirte Klavier* and found that in each case, the notated key correlated significantly with the distribution of tones in the first four notes.

11. Cohen (1977) carried out an experiment in which musicians were asked to identify the key of a prelude after hearing just the first four tones and Krumhansl (1990) found that the algorithm performed better than the subjects in Cohen's (1977) experiment.

8. Krumhansl (1990, Chapter 5): Perceived relations between tones





8. Krumhansl (1990, Chapter 5): Perceived relations between tones

1. To investigate the perceived relationships between tones, Krumhansl used a variant of the probe-tone method.
2. In this experiment, subjects were presented with trials in which they first heard a strong key-defining context followed by two tones, played in succession forming a melodic interval. In each trial, the subject had to rate on a scale of 1 to 7 how well the second tone followed the first in the given context.
3. The graph on the left shows the average rating obtained for each final pitch when preceded by a major key defining context together with the major key profile. As you can see, both graphs go up and down in roughly the same places and in fact the correlation between them is .97 which is very high indeed.
4. The graph on the right shows the average rating obtained for each final pitch when preceded by a minor key defining context together with the minor key profile. The correlation is .92 which is only slightly less than that for the major key contexts.
5. This shows that listeners like to hear intervals that end on tones which have high ratings in the tonal hierarchy of the preceding context.
6. This reinforces the interpretation that the probe-tone ratings obtained in the Krumhansl and Kessler (1982) experiments can be interpreted as being a measure of stability or finality.

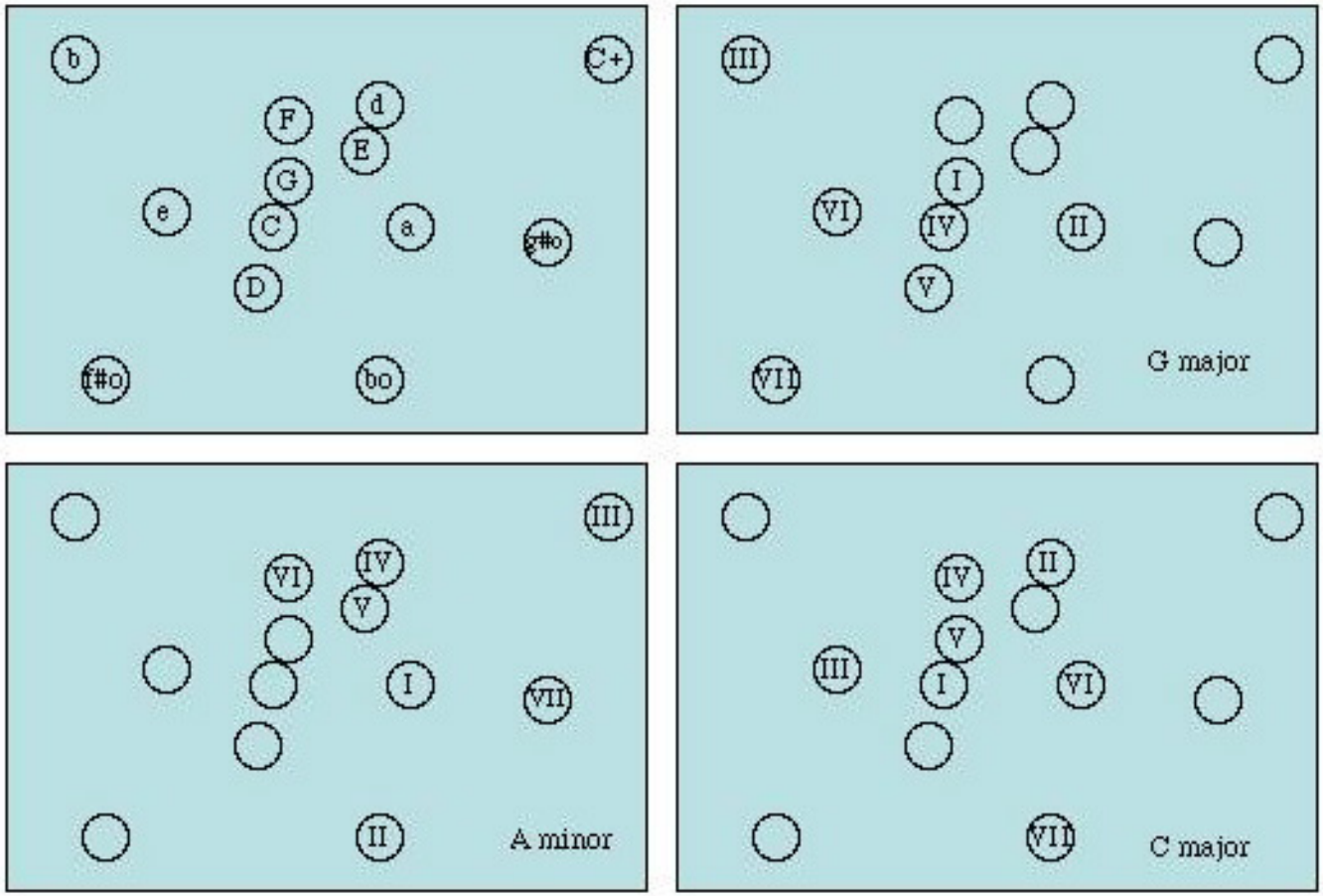
9. Krumhansl (1990, Chapter 7): Perceived relations between chords and keys

- Subjects generally gave higher ratings to major triads than minor triads and higher ratings to minor triads than diminished triads.
- Subjects gave higher ratings to chords that function as basic harmonic elements in the context key.
- Subjects gave higher ratings to chords containing tones that have high ratings in the tonal hierarchy.
- By far the best predictor of the ratings given to the fittingness of chords was the ratings of the component tones of the chords in the tonal hierarchy for the key.

9. Krumhansl (1990, Chapter 7): Perceived relations between chords and keys

1. Krumhansl (1990, Chapter 7) describes two probe-tone-like experiments that she carried out in order to investigate the perceived relatedness between triads and keys.
2. In each trial of the first experiment, subjects were presented with a context consisting of a rising and falling major or minor scale which was then followed by a major, minor or diminished triad. The subjects had to rate how well the triad fit with the preceding context.
3. In the second experiment, the context was changed to a chord progression containing all the chords in the key being defined, ordered along the circle of fifths.
4. The results obtained were very similar for both contexts.
5. Krumhansl (1990, p. 177) derived the following conclusions from analysing the results of these experiments:
   (a) Subjects generally gave higher ratings to major triads than minor triads and higher ratings to minor triads than diminished triads.
   (b) Subjects gave higher ratings to chords that function as basic harmonic elements in the context key.
   (c) Subjects gave higher ratings to chords containing tones that have high ratings in the tonal hierarchy.
   (d) By far the best predictor of the ratings given to the fittingness of chords was the ratings of the component tones of the chords in the tonal hierarchy for the key.

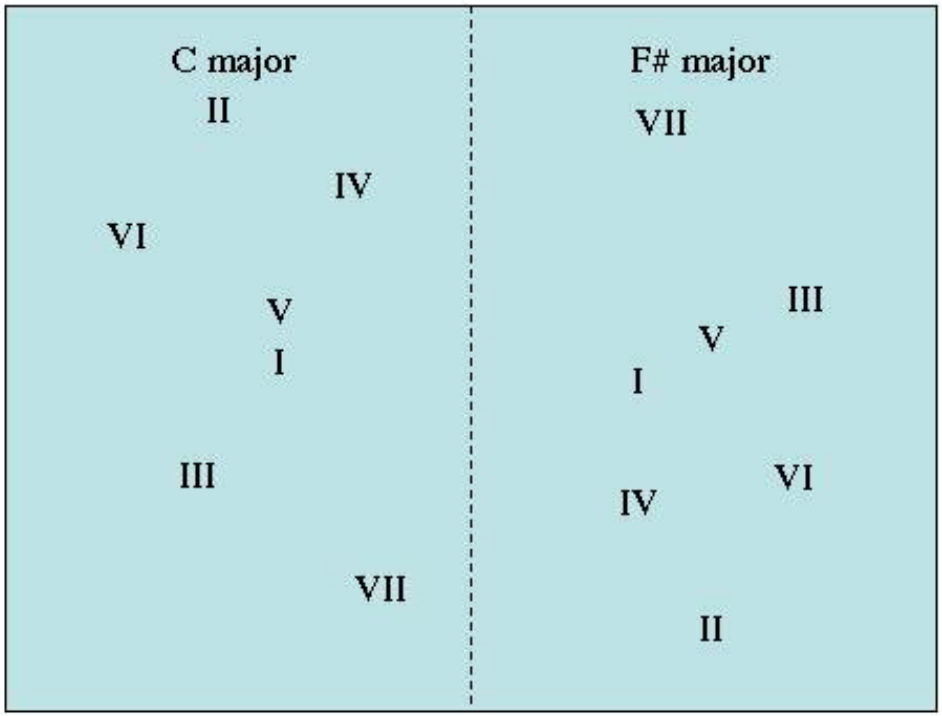
10. Krumhansl, Bharucha, and Kessler (1982b): The perceived relations between chords within keys



10. Krumhansl, Bharucha, and Kessler (1982b): The perceived relations between chords within keys

1. Krumhansl, Bharucha, and Kessler (1982b) carried out an experiment to explore the perceived relatedness between chords in a tonal context.
2. In each trial of the experiment, the listeners were first presented with a key-defining context consisting of a rising scale in either C major, A minor or G major. This was then followed by two chords, each taken from one of the three keys. Listeners had to judge how well the second chord followed the first in the context of the scale sounded at the beginning of each trial.
3. The ratings did not depend in a regular way on the key of the context.
4. They used the multidimensional scaling technique to derive a spatial representation in which chords rated more similar to each other were positioned more closely together in the space. The result is shown here at the top and then repeated three times below to show the positions of the chords within particular keys.
5. Note that the chords in G major emerged on the left-hand side of the space, while the chords of C major emerged in the middle and those in A minor emerged on the right-hand side.
6. So the experiment showed that chords in the same key are perceived as being more closely related than chords in different keys.
7. The fact that the results were independent of the context that began each trial suggested that the perceived distance between chords is essentially independent of the tonal context.

11. Bharucha and Krumhansl (1983): The perceived relations between chords within keys



11. Bharucha and Krumhansl (1983): The perceived relations between chords within keys

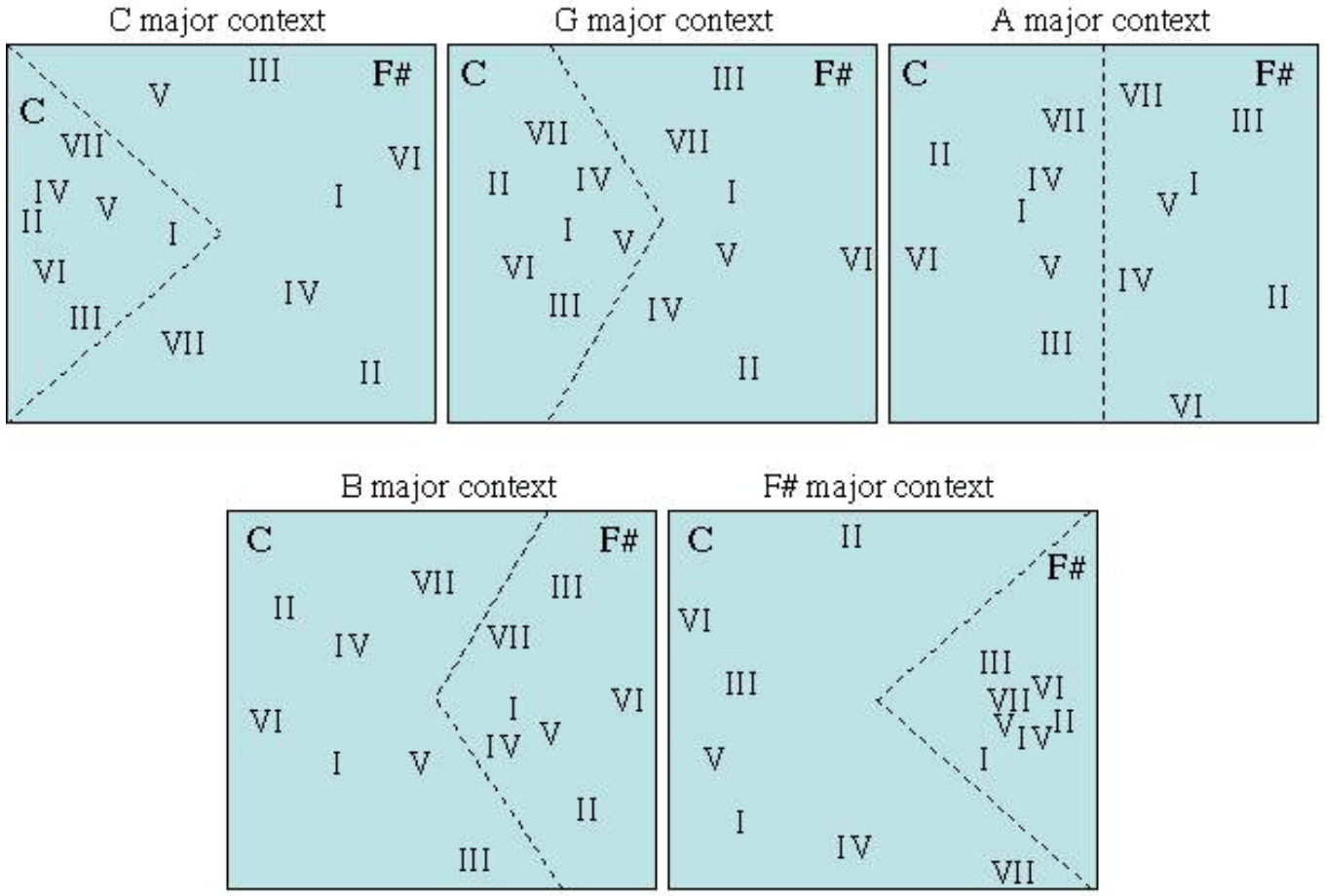
1. Bharucha and Krumhansl (1983) showed that chords in different keys are perceived to be less related than chords within the same key even in the absence of any key-defining context.
2. In this experiment, Bharucha and Krumhansl (1983) investigated the perceived relatedness between the diatonic chords in the keys of C major and F♯ major.
3. According to traditional tonal theory, these two keys are as unrelated as any two keys can be since they are diametrically opposed on the circle of fifths. The two keys have no diatonic triads in common.
4. In Bharucha and Krumhansl's (1983) experiment each participant heard every possible pair of chords that could be formed from the 14 triads in the two keys and for each pair of chords, each participant had to judge how well the second chord followed the first one.
5. In one of the conditions in this experiment, each pair of chords was presented with no preceding context whatsoever and this figure here shows the results of a multidimensional scaling analysis performed on the results obtained under this condition.
6. Krumhansl (1990, pp. 191–192) claims that this analysis shows that "listeners judged the diatonic triads of C major as generally more related to one another than they were to the diatonic triads of F♯ major".
7. However, I don't think that, strictly speaking, these results *do* suggest this since, for example, chord IV in C major is closer to chord VII in F♯ major than it is to chord VII in C major.
8. On the other hand, as Krumhansl (1990, p. 191–192) correctly points out, all the chords in C major are found on the left hand side of the space and all the chords in F♯ major are on the right-hand side of the space. This suggests that the horizontal dimension in this space corresponds to key, which implies that the key to which a chord belongs is one factor in determining how related it is perceived to be to another chord.
9. Another feature of this space which Krumhansl does not remark upon is that if you superimpose an approximate circle onto it, centred in the middle of the diagram, then the corresponding triads in the two keys come out as being

diametrically opposed. For example, II in C major is diametrically opposed to II in F♯ major. This chord-level diametric opposition may be a reflection of the key-level opposition between the two keys on the circle of fifths.

10. Finally, as Krumhansl (1990, p. 191) observes, the primary triads in each key are closely clustered together in the middle of the region occupied by the triads of that key.

11. So, this experiment and the previous one of Krumhansl, Bharucha, and Kessler (1982b) provides experimental evidence that supports the well-established music-theoretical principle that two chords are heard to be more closely related if they can be interpreted as being in the same key or two closely related keys.

12. Bharucha and Krumhansl (1983) and Krumhansl, Bharucha, and Castellano (1982a): The effect of the key of the context on the perceived relatedness between triads



12. Bharucha and Krumhansl (1983) and Krumhansl, Bharucha, and Castellano (1982a): The effect of the key of the context on the perceived relatedness between triads

1. The condition of having no preceding context was just one of the conditions in the experiment reported by Bharucha and Krumhansl (1983).

2. In another condition in this experiment and in a second experiment reported by Krumhansl, Bharucha, and Castellano (1982a), each pair of diatonic chords in the set of 14 triads in the keys of C major and F♯ major was presented preceded by a key-defining context.

3. Each of these key-defining contexts consisted of a IV-V-I chord progression in C major, G major, A major, B major or F♯ major.

4. These keys were chosen because it was hypothesized that the effect of the context key on the judgements of relatedness between the two chords in a trial would depend on the proximity of the context key to the key(s) of the two chords as measured along the circle of fifths.

5. G major and B major were therefore chosen because they are just one step along the circle of fifths away from C major and F♯ major respectively; and A major was chosen because it is equidistant from C major and F♯ major along the circle of fifths.

6. Each of these diagrams shows the multidimensional scaling obtained for the ratings on the chord pairs for a given context key. And, as you can see, the context key was seen to have a regular effect on the ratings obtained. The chords in C major were drawn more closely together in the multidimensional scaling solution when the context key was C major or G major; and the chords of F♯ major were drawn more closely together when the context key was B major or F♯ major.

7. When the context key was A major, half way between C major and F♯ major on the circle of fifths, the chords of C major were spread out over roughly the same area in the space as the chords of F♯ major.

8. This result is an example of what Krumhansl calls the "contextual distance" principle which states that the average perceptual distance between any two elements "varies inversely with the extent that the elements are stable or play significant functions in the context key" (Krumhansl, 1990, p. 196).

9. So a given pair of chords will be perceived to be more closely related if they are presented in the context of a key in which they are stable than if they are presented in the context of a key in which they are unstable or play no significant role.

13. The perception of key change in tonal music (Krumhansl and Kessler, 1982)

	Key	1	2	3	4	5	6	7	8	9
1.	C	F	G	a	F	C	a	d	G	C
2.	c	f	G	A♭	f	c	A♭	d°	G	c
3.	C→G	F	G	C	a	e	b	e	D	G
4.	C→a	F	G	C	F	d	E	b°	E	a
5.	c→f	d°	G	c	A♭	f	D♭	b♭	C	f
6.	c→C	d°	G	c	A♭	G	a	F	G	C
7.	c→A♭	d°	G	c	A♭	f	E♭	b♭	E♭	A♭
8.	C→d	F	G	C	F	d	B♭	e°	A	d
9.	C→B♭	F	G	C	a	F	g	E♭	F	B♭
10.	c→c♯	d°	G	c	G	A♭	A	f♯	g♯	c♯

13. The perception of key change in tonal music (Krumhansl and Kessler, 1982)

1. We're now going to consider an experiment carried out by Krumhansl and Kessler (1982) designed to investigate the 'developing and changing sense of key' (Krumhansl, 1990, p. 214) that we perceive when we listen to Western tonal music.

2. In this experiment, Krumhansl and Kessler (1982) presented listeners with 10 chord sequences, each sequence consisting of 9 chords as shown in this table (Krumhansl, 1990, pp. 214–226).

3. As you can see, the first two sequences were designed to suggest the keys of C major and C minor respectively without any modulation. Sequences 3 to 7 were designed to suggest modulations to closely related keys and sequences 8 to 10 were designed to suggest modulations to more distantly related keys.

4. In the experiment, each subject was presented with the first chord in each sequence, followed by all possible probe tones in the chromatic scale and then asked to rate how well each probe tone fit with each chord. This generated a rating profile for each sequence for a context consisting of the first chord in the sequence.

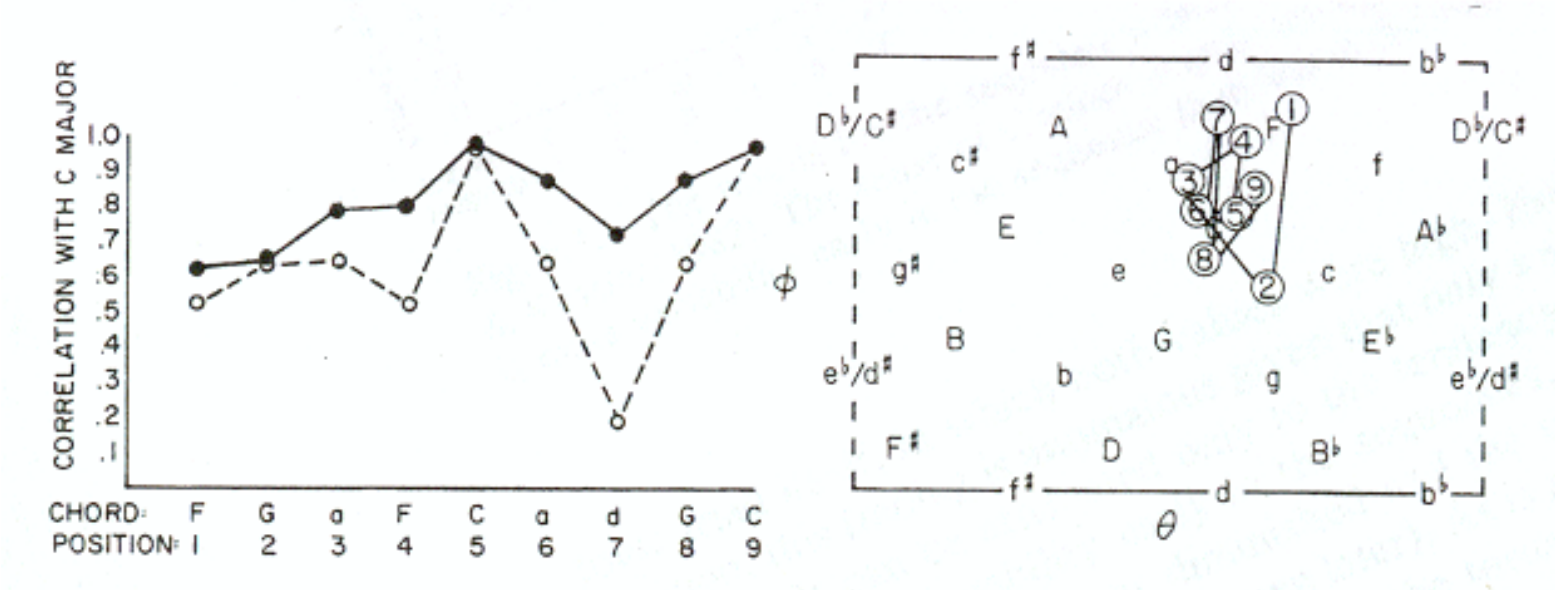
5. Then each subject was presented with the first *two* chords in each sequence as a context, again followed by all possible probe tones from the chromatic scale and again they had to rate the fittingness of each probe tone with each two-chord context. This generated a rating profile for the two-chord prefix of each of the ten sequences.

6. This process was continued until rating profiles had been generated for all prefixes of all the sequences up to and including the complete sequences.

7. All the participants had at least a moderate amount of musical training and there was strong agreement between their responses. Krumhansl and Kessler (1982) therefore found the average profile for each context.

8. Having found all these average profiles, Krumhansl and Kessler (1982) then found the correlation between each average profile and each of the 24 tonal hierarchy profiles for the major and minor keys. This produced a vector of 24 numbers for each context describing the degree to which each of the 24 major and minor keys was implied at the end of that context.

14. The perception of key change in tonal music (Krumhansl and Kessler, 1982)



14. The perception of key change in tonal music (Krumhansl and Kessler, 1982)

1. So by this stage in the analysis, Krumhansl and Kessler (1982) could say how strongly each key was implied at each point in each of the 10 chord sequences.

2. This meant that they could draw a graph like this one here on the left in which the solid line shows the strength with which the key of C major was perceived to be implied after each of the 9 chords in the first sequence.

3. As you can see, the key of C major becomes more and more strongly implied as the chord sequence progresses. In particular, note that after hearing the second G major triad at position 8, the key of C major is more strongly implied than it is after the first time that the G major triad is sounded at position 2. A similar effect can be observed for the A minor and F major triads.

4. The dashed line shows how much the key of C major would be implied if each chord were presented in isolation. For example, the open circle for position 3 indicates that the correlation between the probe tone rating profile for a single A minor triad context and the tonal hierarchy profile for C major is a little more than .6.

5. The important thing to note in relation to the dashed line is that it never goes above the solid line. In other words, the strength with which C major is implied at each point in the first chord sequence is never less than the amount that it would be implied if only the last chord in the preceding context were heard in isolation.

6. Recall that Krumhansl and Kessler (1982) found that the inter-key distances between the keys implied by the correlations between their probe-tone rating profiles could be represented by plotting the keys in a regular pattern over the surface of a torus.

7. Now remember I said that Krumhansl and Kessler (1982) found for each position in each of the 10 chord sequences a 24-number vector indicating the relative strength of implication of each possible key at that point in that sequence.

8. Let's assume that one of these 24-number vectors has a zero in all positions except the one corresponding to A major. This vector would obviously correspond to the position in this toroidal key space where A major occurs.

9. Now let's assume that another one of these 24-number vectors contains a zero in all positions but values of .5 for A major and .5 for E minor. This vector would correspond to the position on the key torus that is exactly half way between A major and E minor, indicating that these two keys are equally strongly implied by the context.

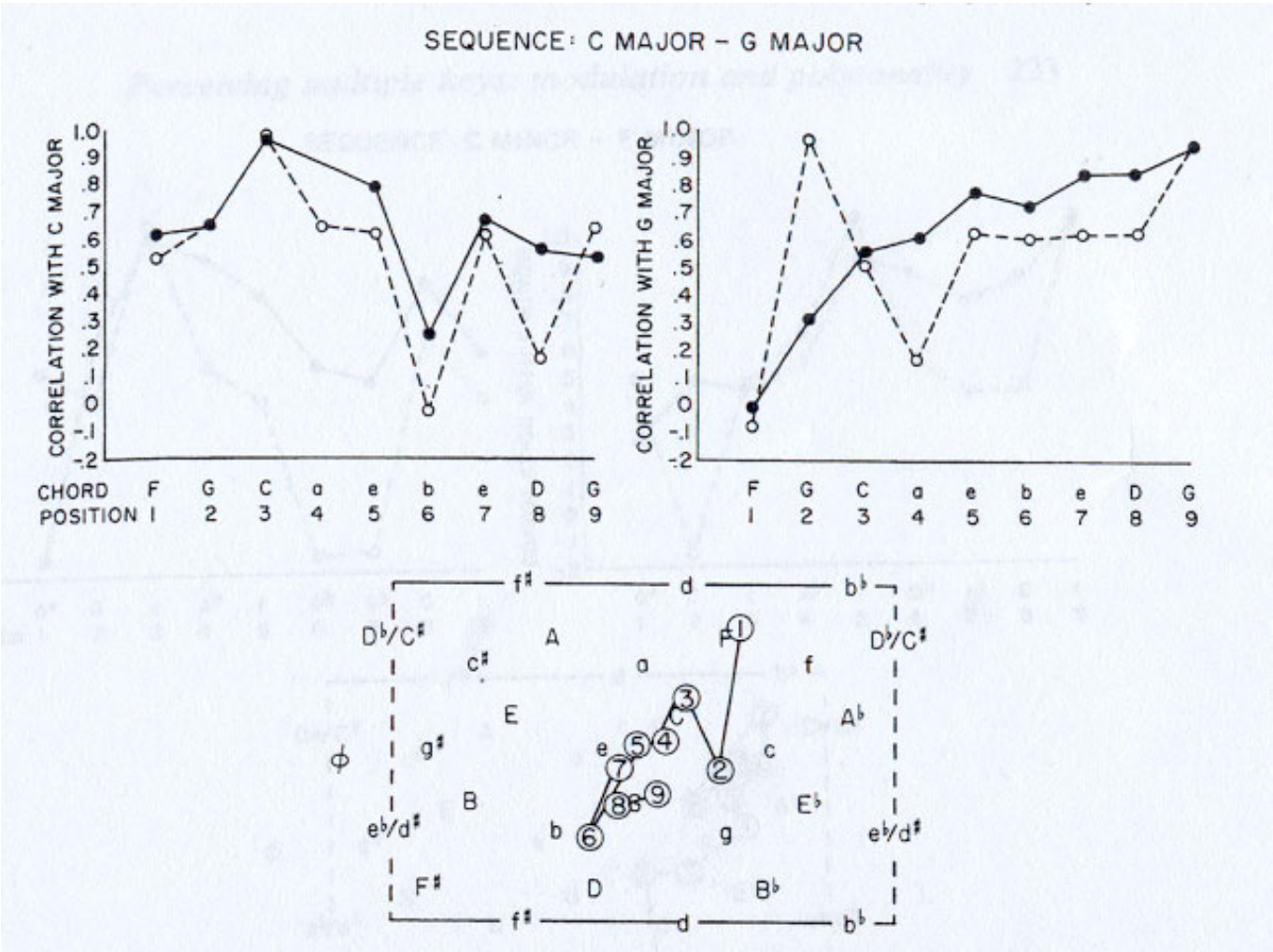
10. Similarly, if the vector contained zeros in all positions except those corresponding to A major, E major and E minor, then this would correspond to a position on the torus somewhere in the triangle formed by these three keys.

11. So you should be able to see that each of the 24-number vectors corresponds to a position in this toroidal key space that expresses the "relative strength of each possible key interpretation for [a particular] position in [a particular] chord sequence" (Krumhansl, 1990, p. 219).

12. This diagram here on the right shows the results of plotting each of these 24-number vectors on the key torus for each position in the first chord sequence. This gives a trace or *multidimensional unfolding analysis* that shows how the sense of key is perceived to change over the course of this chord sequence.

13. As you can see, F major is strongly implied at the beginning of the sequence (because the first chord is an F major triad) but then the key spirals in on C major.

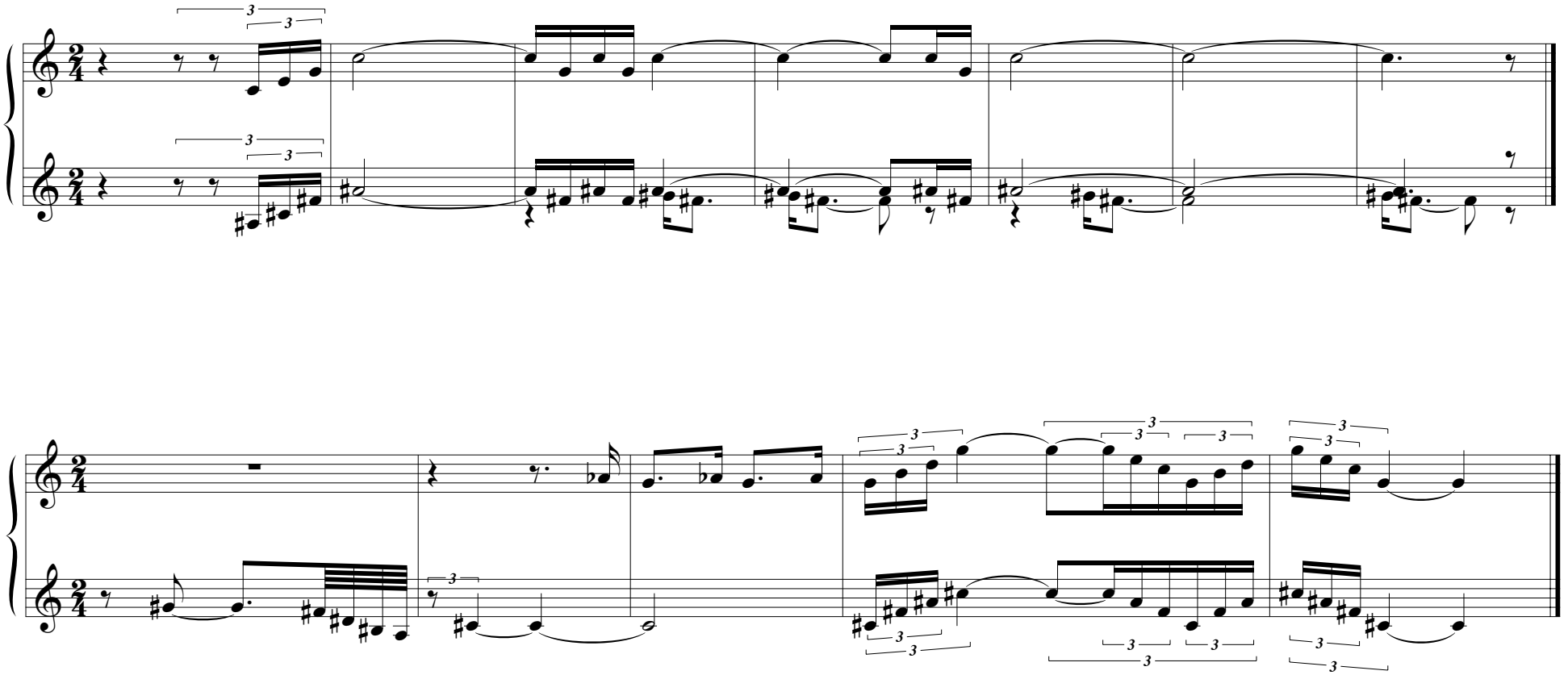
15. The perception of key change in tonal music (Krumhansl and Kessler, 1982)



15. The perception of key change in tonal music (Krumhansl and Kessler, 1982)

1. These graphs show the corresponding results for the sequence that was designed to be perceived to modulate from C major to G major.
2. As you can see, as the chord sequence progresses, the strength which which C major is implied generally falls and the strength which which G major is implied rises steadily.
3. Similarly, in the multidimensional unfolding analysis, the key centre moves from F major for the first chord down to C major. Then over the second half of the sequence it moves to G major.
4. So, there are at least three important things to note about this experiment.
   (a) The probe-tone method can be used in conjunction with the spatial representation of keys to produce a picture of how a listener's perceived sense of key changes over the course of a passage.
   (b) The perceived key at a particular point in the music depends not just on the event that one has just heard but on the sequence of events heard up to that point.
   (c) The sense of key at a particular point in a passage is usually not completely definite and unambiguous even when the passage consists of diatonic triads. Instead, typically two or more keys are implied to some extent but with different relative strengths.

16. The perception of polytonality (Krumhansl and Schmuckler, 1986)



16. The perception of polytonality (Krumhansl and Schmuckler, 1986)

1. We've just seen how the sense of key changes over time as one listens to a passage of tonal music and we saw that often two or more keys may be implied at the same time.

2. Now we consider the extent to which a listener may hear two or more keys simultaneously: a phenomenon known in music theory as *polytonality*.

3. Krumhansl and Schmuckler (1986) explored the perception of polytonality by investigating the way that listeners responded to this passage from Stravinsky's *Petroushka*.

4. As you can see, the first section consists of an arpeggiated C major triad played simultaneously with an arpeggiated F♯ major triad, each triad being presented with essentially the same rhythm.

5. In the second section, there are again basically two streams, one in C and the other in F♯ major.

6. Note that in both sections of the passage, both streams are presented within the same pitch range and both streams have approximately the same rhythm and contour.

7. Krumhansl and Schmuckler (1986) carried out a number of different experiments in which the passage was presented to listeners under a variety of different conditions.

8. First they presented the complete passage to listeners with the C and F♯ streams combined and got the participants to rate how well each of the 12 tones in the chromatic scale followed this context. Then they did the same thing with each of the two streams presented separately.

9. They found that the probe-tone ratings obtained when the C stream was presented on its own correlated very strongly with the tonal-hierarchy profile for the key of C major. Similarly, they found that the probe-tone ratings obtained when the F♯ major stream was presented on its own correlated very strongly with the tonal-hierarchy profile for the key of F♯ major. In other words, when each stream was presented in isolation it strongly implied a single key.

10. When both streams were presented simultaneously, the probe-tone ratings obtained correlated very well with the weighted average of the tonal hierarchies for C major and F♯ major.

11. As you saw earlier on, the tonal hierarchies were shown to correlate very well with the frequency distributions of pitch classes in tonal music. Krumhansl and Schmuckler (1986) therefore hypothesized that the probe-tone ratings obtained for the *Petroushka* passage might correlate well with the frequency distributions of the pitch classes within the passage.

12. They found that this was indeed the case both when the two streams were presented together and when they were presented separately.

13. In other words, the more often a particular pitch class occurred in the passage, the better it was considered to fit with the passage.

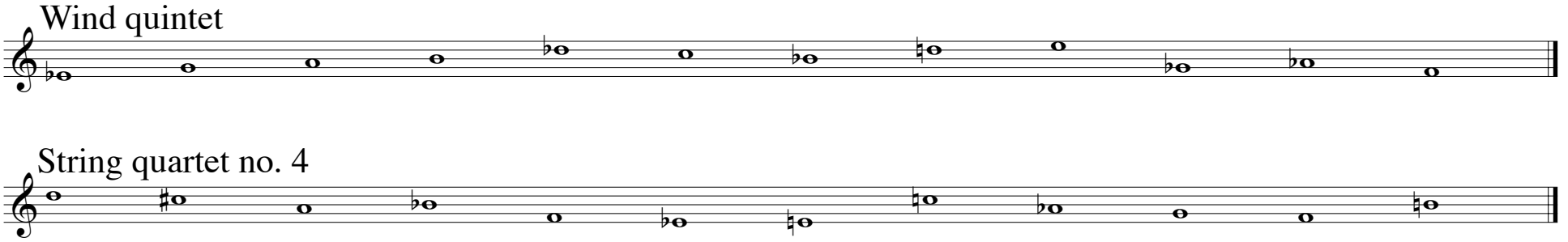
14. However, Krumhansl and Schmuckler (1986) found that they could predict the probe-tone ratings obtained when the two streams were presented separately, significantly more accurately if they combined the frequency distribution data for the passage with the tonal hierarchy profile for the key of C major.

15. What this suggests is that when a listener heard one of the streams on its own, the perception of how well a particular pitch class "fit" with the passage was determined by the relative frequency with which that pitch class occurred in the passage *and* the listener's previously internalized tonal hierarchy for the perceived key of that stream.

16. On the other hand, when the complete passage was presented with the two streams together, Krumhansl and Schmuckler (1986) found that the probe-tone ratings obtained were predicted better by considering just the frequency distributions of the pitch classes within the passage than by combining the key profiles for C and F♯ major with these frequency distributions.

17. This seems to suggest that when the listeners heard the combined passage, no strong sense of key was induced so the relative stability or "fittingness" of each pitch class was determined by the "bottom-up" relative frequency distribution of the pitch classes within the passage.

17. Tonal hierarchies in serial atonal music (Krumhansl, Sandell, and Sergeant, 1987)



- Group 1 listeners gave higher ratings to tones not in the context and tones that conflicted with implied key.
- Group 2 listeners gave higher ratings to tones in context and highest ratings to most recent tones.

17. Tonal hierarchies in serial atonal music (Krumhansl, Sandell, and Sergeant, 1987)

1. Krumhansl, Sandell, and Sergeant (1987) carried out a series of experiments designed to investigate whether listeners use tonal hierarchies when listening to atonal serial music such as that of the so-called "Second Viennese School".

2. In the first of these experiments, listeners were presented in each trial with the first 3, 6, 9 or 12 tones in one of the 12-tone rows shown here, followed by one of the 12 tones in the chromatic scale.

3. The upper row is the one used in Schoenberg's Wind Quintet (Op. 26) and the lower row is the one that he uses in his Fourth String Quartet (Op. 37).

4. Each listener had to rate the fittingness of each of the possible probe-tones to each of the possible contexts *twice*. This replication was carried out in order to test how consistently the listeners were responding and to determine if the responses given were affected by having heard the context once already.

5. The results showed that the participants could be neatly divided into two groups, which we'll refer to as Group 1 and Group 2.

6. The listeners in Group 1 had had more experience of atonal music than those in Group 2. Group 1 listeners generally gave higher ratings to tones that were *not* present in the context and lower ratings to tones that *were* present in the context.

7. In other words, the responses of the Group 1 listeners were consistent with the principle of serial technique that no tone should be repeated until all the other tones have been sounded.

8. The results also showed that the Group 1 listeners did not give higher ratings to the tones immediately following incomplete contexts. For example, they did not in general rate the pitch class of the fourth tone particularly highly in the context of the first three tones in a row. This was the case even on the second exposure to a particular context-probe-tone combination.

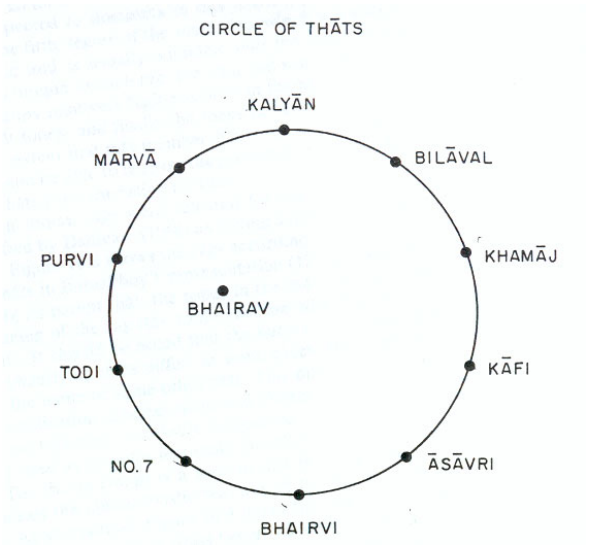
9. In other words, although the Group 1 listeners could fairly reliably tell if a probe-tone had already occurred within a context, they were not able to remember the precise ordering of the tones in the rows.

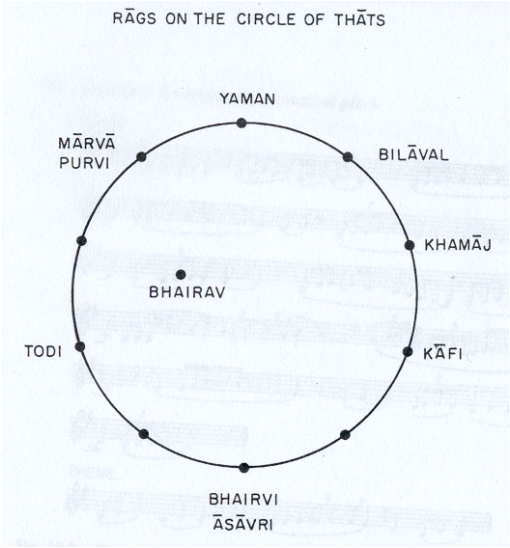
10. Further analysis of the results showed that Group 1 listeners consistently gave lower ratings to tones that more strongly implied any particular key. In other words, they seemed to be using the internalized tonal hierarchies that they had acquired through their exposure to tonal music *in the opposite way* to that in which they used these hierarchies when listening to tonal music.

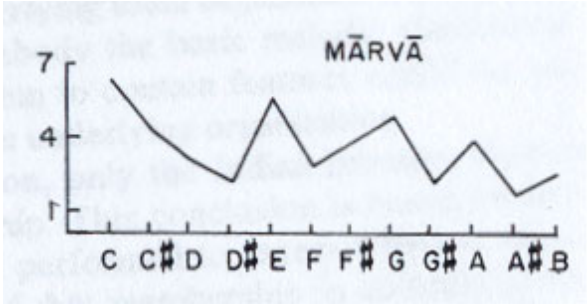
11. That is, they were responding to the atonal contexts as though they were *anti-tonal*.

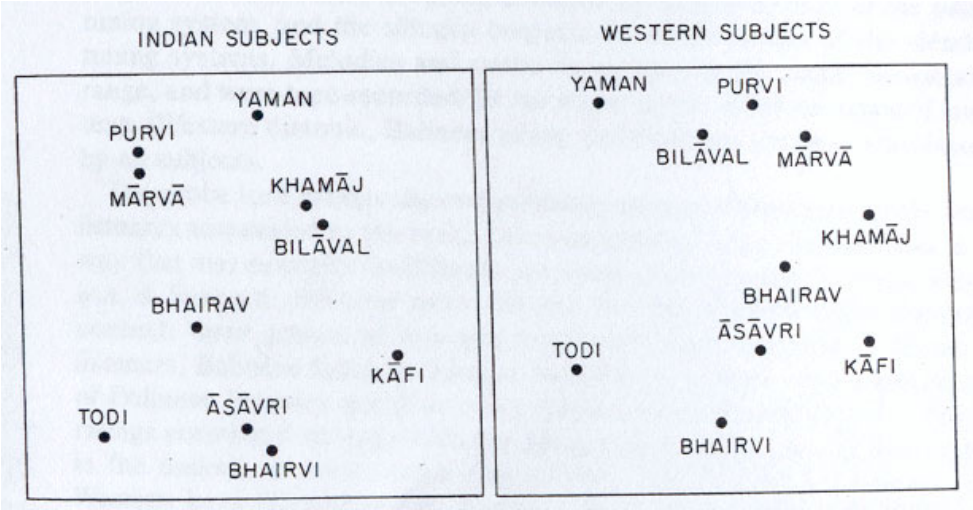
12. The probe-tone ratings obtained from the listeners in Group 2 indicated that these listeners were using essentially the opposite strategy in their responses: pitch classes that had already occurred within the context were rated more highly than those that had not occurred with the highest ratings being given to those tones that had occurred most recently.

18. Tonal hierarchies in North Indian classical music (Castellano, Bharucha, and Krumhansl, 1984)









18. Tonal hierarchies in North Indian classical music (Castellano, Bharucha, and Krumhansl, 1984)

1. Finally, I'd like to acquaint you with an experiment in which the probe-tone method was used to investigate pitch structure perception in non-Western music.
2. Castellano, Bharucha, and Krumhansl (1984) carried out an experiment in which both Indian and Western listeners were required to rate the fittingness of probe-tones (chosen from the equal-tempered chromatic scale) with contexts consisting of extracts from North Indian rāgs.
3. Each rāg can be thought of as being essentially a set of characteristic melodic features, including, for example, a typical catch-phrase, ascending line and descending line.
4. Also, each rāg is (at least in theory) based on a particular 7-note scale called a thāt.
5. As for the Western tonal scales, the first and fifth degrees of each thāt, called Sa and Pa respectively, have particular importance and Pa is tuned to be approximately 3/2 times the frequency of Sa.
6. In theory, each of the other 5 degrees of a thāt takes one of two values giving a total of 32 possible thāts. However, only about 10 of these are in common use.
7. In Indian music theory, it is customary to represent the relatedness or similarity between these 10 thāts by placing them on a circle as shown here on the left. Nine of the thāts fall on the circumference of the circle and the tenth, thāt Bhairav, is positioned inside the circle as shown. Thāt No. 7 is not in common use but it is located on the circle of thāts.
8. If we assume that the pitch of Sa is the same for all the thāts on this circle, then any two neighbouring thāts on this circle differ by just one pitch.
9. For each rāg, two pitches, called vādi and samvādi, play a special role. These two tones are usually either a fifth or a fourth apart and they are usually different from the Sa and Pa of the underlying thāt. The vādi pitch usually plays a more important role than the samvādi pitch as it typically occurs in prominent positions and is usually repeated more often than samvādi.

10. Castellano, Bharucha, and Krumhansl (1984) chose 10 rāgs for their experiment and this diagram here shows, for each rāg, the thāt on which it was based.

11. In each trial in the experiment, a listener heard an extract from one of these rāgs followed by a probe-tone from the chromatic scale. Each listener heard all context-probe-tone combinations that could be formed from the 10 rāg contexts and 12 probe-tones and, for each combination, had to rate how well the probe-tone fit with the context.

12. Castellano, Bharucha, and Krumhansl (1984) used two groups of participants. One group consisted of students who had had formal training in Indian music and the other consisted of students who had had about the same amount of musical training but who had not had any exposure to Indian music.

13. These two groups of listeners produced strikingly similar responses so Castellano, Bharucha, and Krumhansl (1984) produced an average rating profile for each rāg. This graph here, for example, shows the average rating profile obtained for rāg Mārvā.

14. When the results were averaged over all rāgs and all participants, the experimenters found that, in general,

    (a) tones in the underlying thāt of the rāg received higher ratings than non-thāt tones; and

    (b) amongst the thāt tones, Sa was generally given the highest rating, followed by Pa and then vādi.

15. For example, in rāg Mārvā, shown here, the highest ratings are given to Sa (C), Pa (G), vādi (E) and samvādi (A).

16. These results therefore accorded well with the predictions of Indian music theory.

17. Recall that Krumhansl (1990) found that there was a very high correlation between the frequency of occurrence of a pitch class (relative to its tonic) in tonal music and its probe-tone rating in the key profile.

18. Castellano, Bharucha, and Krumhansl (1984) similarly found that there was a very high correlation between the sum of the durations of a given pitch class in a given rāg extract and its average probe-tone rating for that rāg.

19. In other words, how well a particular pitch class fit with a given rāg extract could be well predicted by the total amount of time that that pitch class was sounding in the extract.

20. However, further analysis of the results showed that the ratings given to the tones by the Indian listeners were also influenced by their knowledge of the underlying thāts, whereas the ratings given by the Western listeners seemed to be entirely governed by tone duration.

21. Finally, recall that Krumhansl and Kessler (1982) managed to show that the perceived relatedness between the Western tonal keys could be well represented by plotting the keys in a regular pattern over the surface of a torus so that the distance between any two keys measured within the surface of the torus was inversely proportional to the correlation between their probe-tone rating profiles.

22. Castellano, Bharucha, and Krumhansl (1984) performed a similar analysis on their results for the rāgs. First they computed the correlation between each pair of rating profiles obtained in their experiment and then they performed a multidimensional scaling on these correlations.

23. This diagram here shows the multidimensional scaling solution obtained for the Indian listeners and this diagram shows the solution obtained for the Western listeners. As you can see, the scaling solution obtained for the Indian listeners is impressively similar to the circle of thāts diagram used in Indian music theory. On the other hand, the scaling solution obtained for the Western listeners bears little resemblance to the circle of thāts.

24. This clearly demonstrates that the Indian listeners' ratings were affected by their knowledge of the underlying music theory.

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